M2 JUNE 02

1) $v = (3t-2)i - 5tj$

a) $a = \frac{dv}{dt} = 3i - 5j$

b) $S = \int v\,dt = \left(\frac{3}{2}t^2 - 2t + C_1\right)i - \left(\frac{5}{2}t^2 + C_2\right)j$

$t = 0, S = 3i \Rightarrow C_1 = 3, C_2 = 0$

$t = 2 \quad S = 5i + 10j \Rightarrow \text{dist} = \sqrt{5^2 + 10^2} = \underline{5\sqrt{5}\,m}$

2) $t = 3 \qquad a = 4t - t^2$

$v = \int a\,dt = 2t^2 - \frac{1}{3}t^3 + C$

$t = 0, v = 0 \Rightarrow C = 0 \qquad v = 2t^2 - \frac{1}{3}t^3$

$t = 3, \ v = \underline{9\ ms^{-1}}$

b) $t = 6 \qquad a = \frac{27}{t^2} \qquad v = \int a\,dt = \int 27t^{-2}\,dt$

$v = -27t^{-1} + C \Rightarrow v = -\frac{27}{t} + C$

$t = 3, v = 9 \Rightarrow 9 = -\frac{27}{3} + C \Rightarrow C = 18$

$\Rightarrow v = 18 - \frac{27}{t} \qquad t = 6 \Rightarrow v = \underline{13.5\ ms^{-1}}$

3) $KE_A + PE_A + \text{Wd by Cyclist} - \text{Wd against Res} = KE_B + PE_B$

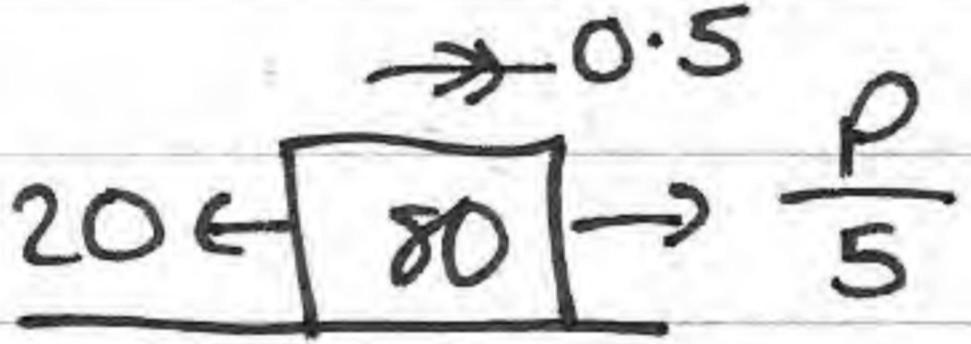
$\therefore$ loss in KE + loss in PE + Wd by Cyclist = Wd against Res

$\frac{1}{2}(80)(8^2 - 5^2) + 80g(20-12) + \text{Wd by C} = 20 \times 500$

$\Rightarrow 1560 + 6272 + \text{wd by } C = 10000$

$\therefore \text{wd by } C = 2168\,J$

b)

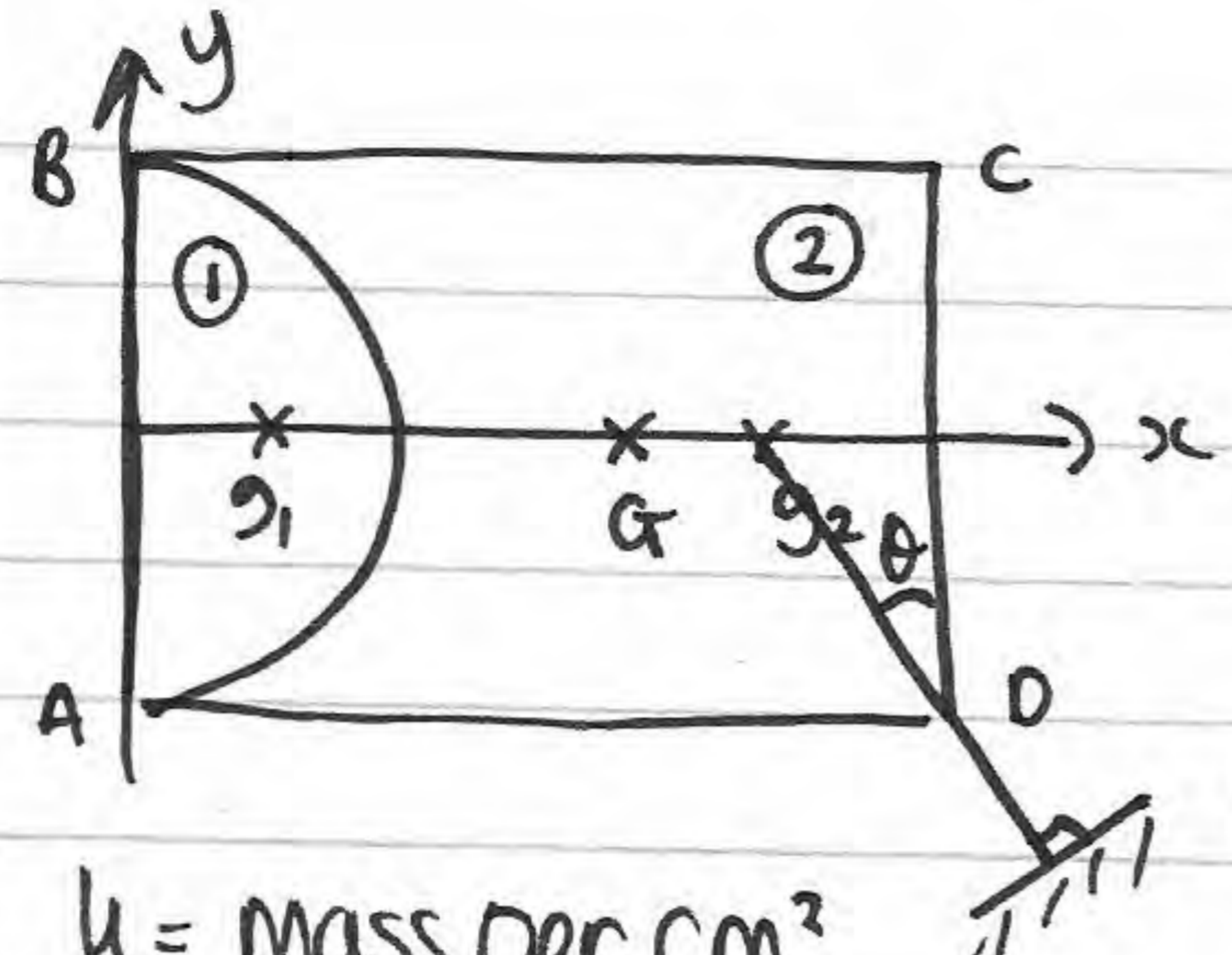


$\vec{Rf} = ma$

$\frac{P}{5} - 20 = 80 \times 0.5$

$\Rightarrow \frac{P}{5} = 60$

$\Rightarrow \underline{P = 300W}$

4) 

$M_1 = \frac{25}{2}\pi u \quad g_1 = \left(\frac{20}{3\pi}, 0\right)$

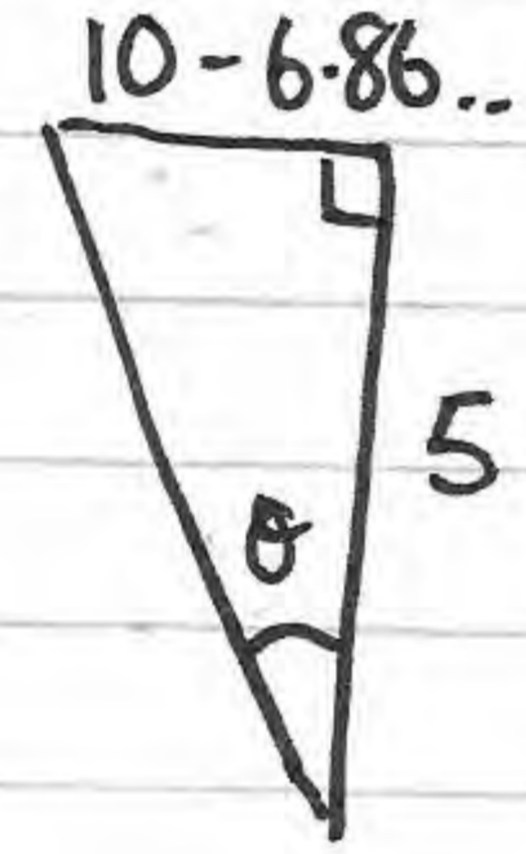
$M_2 = \left(100 - \frac{25}{2}\pi\right)u \quad g_2 = (\bar{x}, 0)$

$M = 100u \quad G(5,0)$

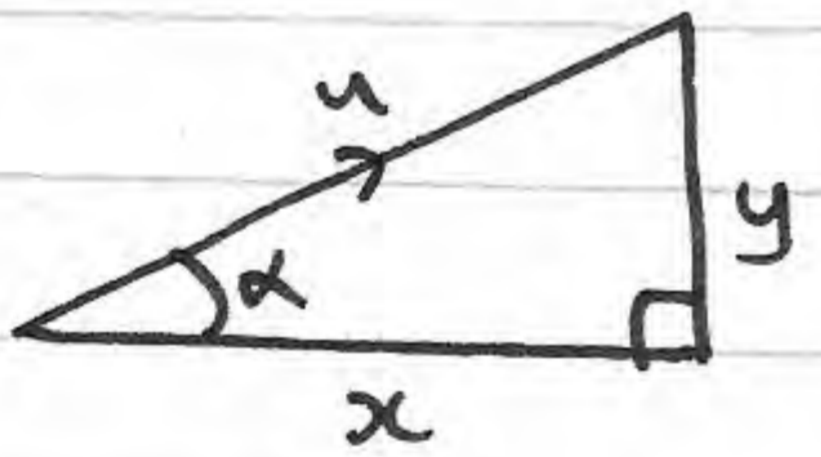
u = mass per cm²

$\frac{25}{2}\cancel{\pi}\cancel{ug} \times \frac{20}{3\cancel{\pi}} + \left(100 - \frac{25}{2}\pi\right)\cancel{ug} \times \bar{x} = 100\cancel{ug} \times 5$

$\Rightarrow \frac{250}{3} + \left(100 - \frac{25}{2}\pi\right)\bar{x} = 500 \quad \Rightarrow \bar{x} = \underline{6.86\ (3sf)}$

b) 

$\theta = \tan^{-1}\left(\frac{10 - 6.86..}{5}\right) \Rightarrow \theta = \underline{32.1^\circ}$

5) 

$\tan\alpha = \frac{y}{x}$

$\overrightarrow{H}$ Vel $= u\cos\alpha \quad x = s$

$x = u\cos\alpha \times t$

$V\uparrow$ $\uparrow u = u\sin\alpha$

$\uparrow a = -9.8$

$\uparrow s = y$

$\Rightarrow y = u\sin\alpha \times t - 4.9t^2$

$t = \frac{x}{u\cos\alpha}$

$\therefore y = \frac{\cancel{u}\sin\alpha \times x}{\cancel{u}\cos\alpha} - \frac{4.9x^2}{u^2\cos^2\alpha}$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\Rightarrow y = x\tan\alpha - \frac{gx^2}{2u^2} \times \sec^2\alpha$

$\tan^2 + 1 = \sec^2$

$\therefore y = x\tan\alpha - \frac{gx^2}{2u^2}(\tan^2\alpha + 1)$

b) $\alpha = 45 \Rightarrow \tan\alpha = 1$ $\qquad -2 = x - \dfrac{9.8x^2}{\cancel{2}(14^2)}(\cancel{1^2}+1)$

$y = -2$

$u = 14$

$\Rightarrow -2 = x - \dfrac{x^2}{20} \quad \Rightarrow \quad x^2 - 20x - 40 = 0$

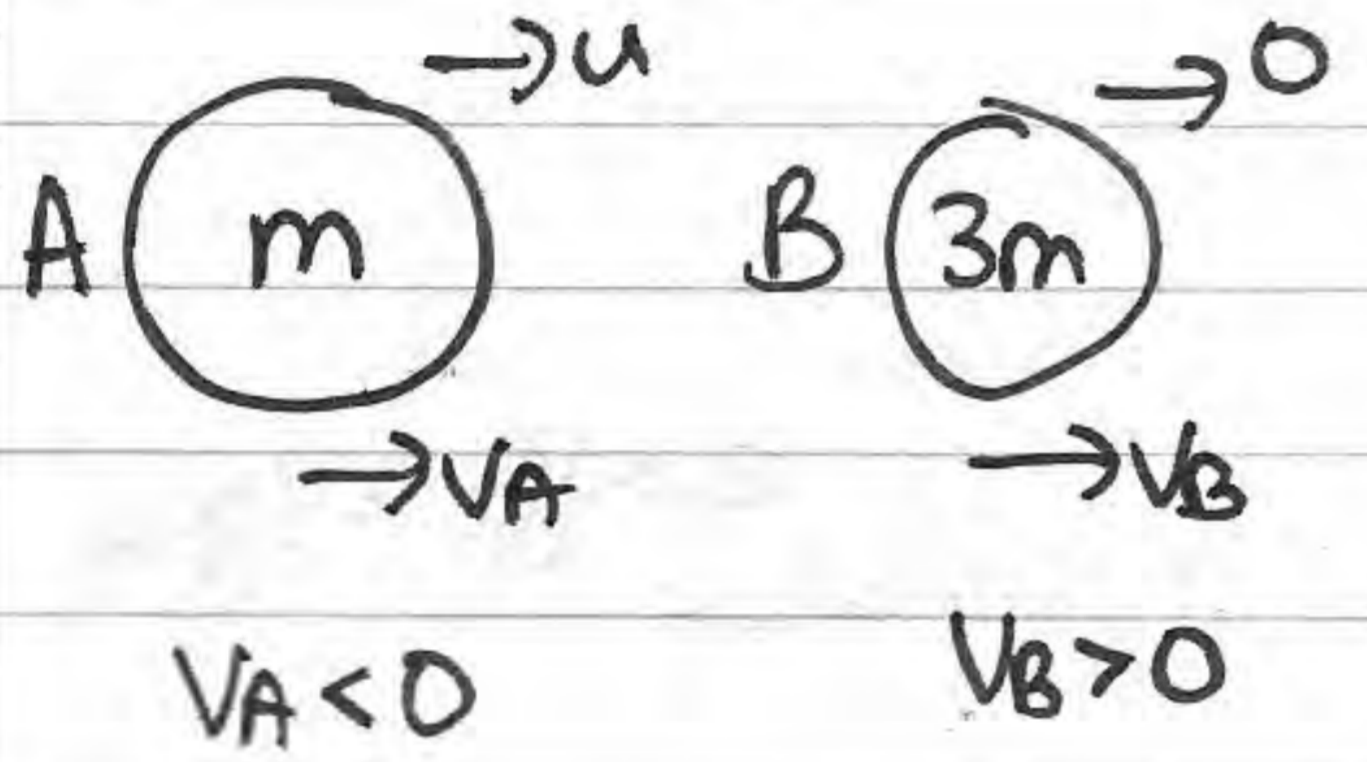
$\times 20$

$\Rightarrow x \Rightarrow (x-10)^2 - 100 = 40$

$\Rightarrow x - 10 = \sqrt{140} \Rightarrow x = 10 + \sqrt{140} = \underline{21.8\,m}$ (3sf)

c) $t = \dfrac{x}{u\cos\alpha} = \dfrac{21.8\ldots}{14\cos 45} \Rightarrow t = \underline{2.2 \text{ sec}}$ (2sf)

6)



$e = \dfrac{V_B - V_A}{u} \Rightarrow eu = V_B - V_A$

$V_B = eu + V_A$

$CLM = \cancel{m}u = \cancel{m}V_A + 3\cancel{m}V_B$

$\Rightarrow u = V_A + 3eu + 3V_A$

$\Rightarrow 4V_A = u - 3eu$

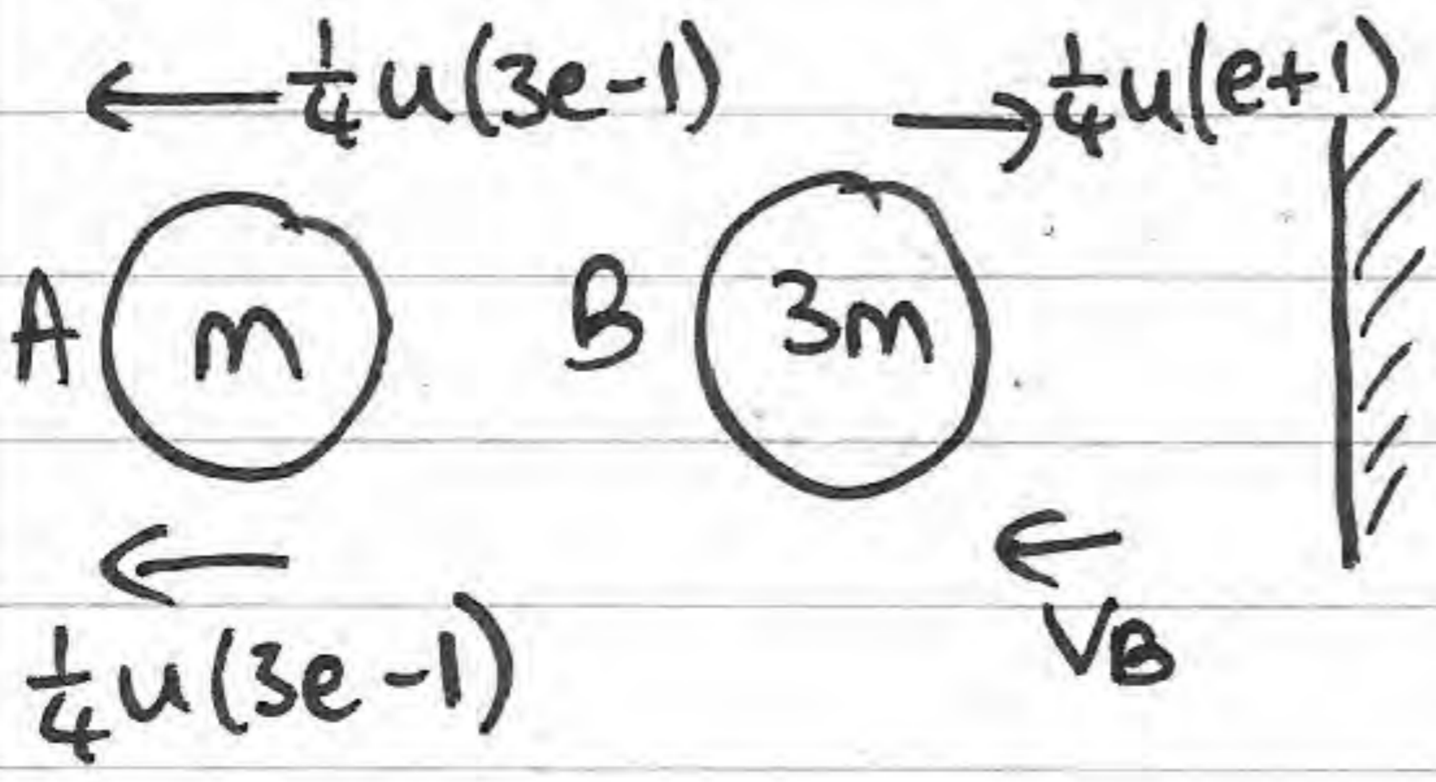
$\Rightarrow V_A = \frac{1}{4}u(1-3e)$

Since $V_A < 0 \Rightarrow 3e > 1 \Rightarrow e > \frac{1}{3}$

$V_B = eu + \frac{1}{4}u - \frac{3}{4}eu$

$\Rightarrow V_B = \frac{1}{4}eu + \frac{1}{4}u \Rightarrow V_B = \frac{1}{4}u(e+1)$

Speed $\overleftarrow{A} = \frac{1}{4}u(3e-1)$ $\qquad$ Speed $\overrightarrow{B} = \frac{1}{4}u(e+1)$



$e = \dfrac{4V_B}{\cancel{\frac{1}{4}}u(e+1)} = \dfrac{3}{4}$

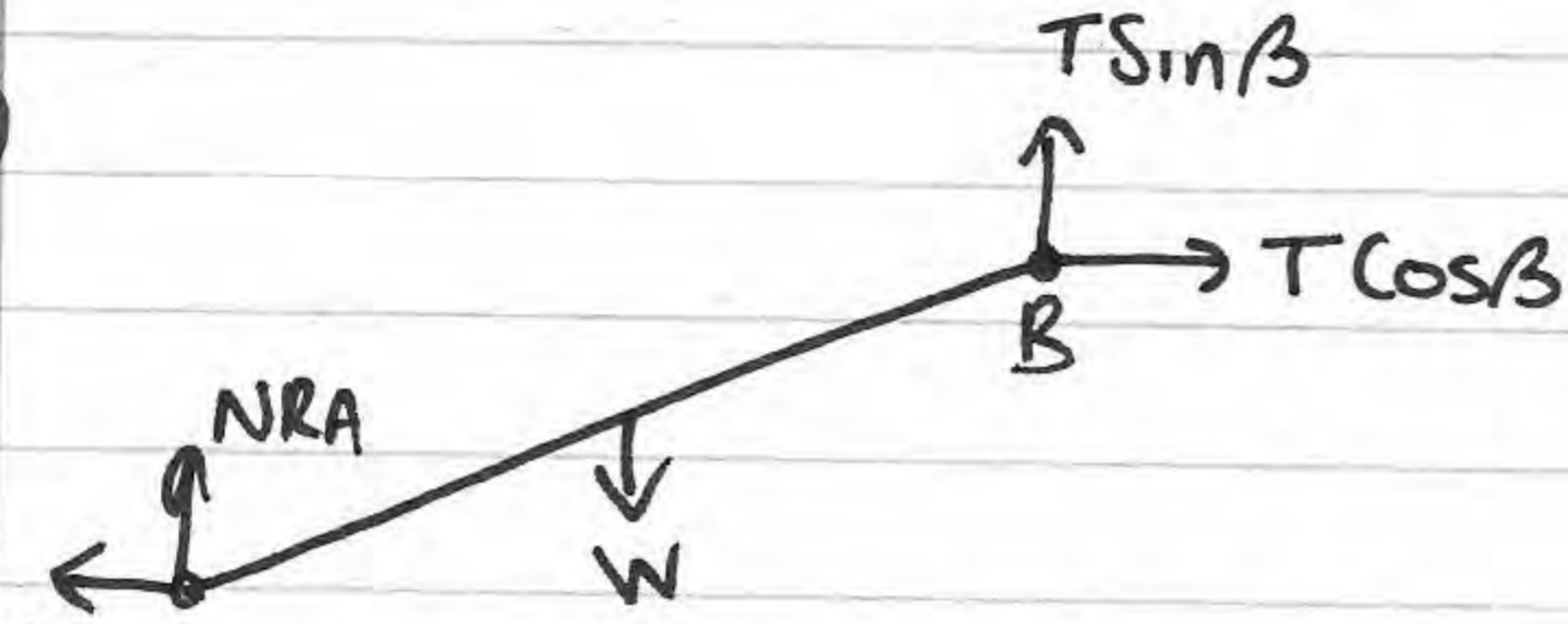
$V_B = \dfrac{3u(e+1)}{16}$

Second collison $\Rightarrow \frac{3\cancel{u}(e+1)}{16} > \frac{1}{4}\cancel{u}(3e-1)$

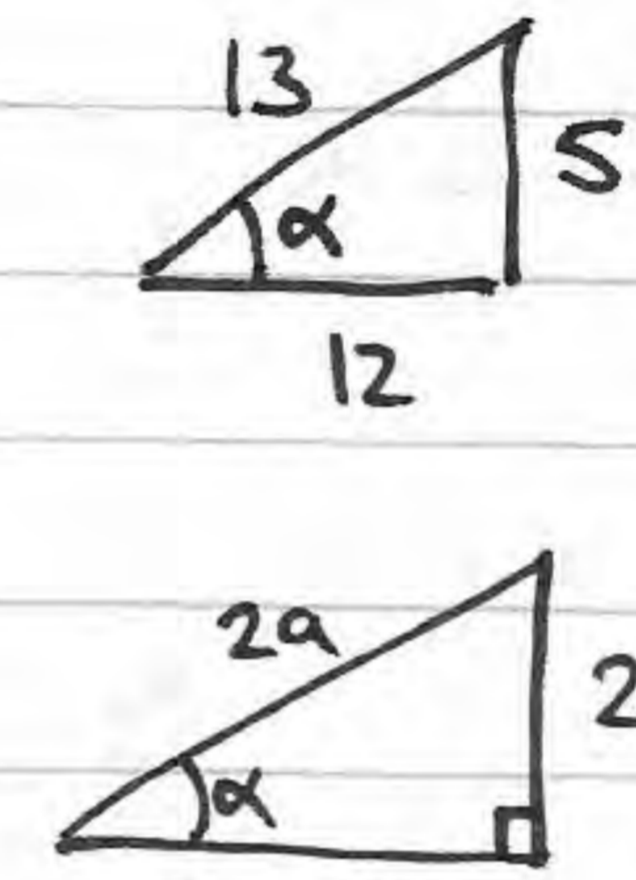
$\Rightarrow 3e+3 > 12e-4 \quad \Rightarrow 7 > 9e \quad \Rightarrow e < \frac{7}{9}$

from (a) $V_A < 0 \Rightarrow e > \frac{1}{3} \quad \therefore \quad \frac{1}{3} < e < \frac{7}{9}$

7)



$f_{max} = \mu NR_A \qquad \mu = 0.6 = \frac{3}{5}$



$\tan\alpha = \frac{5}{12}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

$2a \times \frac{5}{13} = \frac{10}{13}a$

$2a \times \frac{12}{13} = \frac{24}{13}a$

$B \circlearrowright$

$W \times a\cos\alpha = NRA \times 2a\sin\alpha + f_{max} \times 2a\cos\alpha$

$\Rightarrow \frac{12}{\cancel{13}}\cancel{a}W = NR_A \times \frac{24}{\cancel{13}}\cancel{a} + \frac{3}{5}NRA \times \frac{10}{\cancel{13}}\cancel{a}$

$\Rightarrow 12W = 24NRA + 6NRA$

$\Rightarrow 12W = 30NRA \quad \therefore NRA = \frac{2}{5}W$ #

b) $R\overrightarrow{f} = 0 \Rightarrow f_{max} = T\cos\beta \Rightarrow T\cos\beta = \frac{3}{5}NRA$

$Rf\uparrow = 0 \Rightarrow T\sin\beta + NRA = W \Rightarrow T\sin\beta = \frac{5}{2}NRA - NRA$

$\therefore \frac{\cancel{T}\sin\beta}{\cancel{T}\cos\beta} = \frac{\frac{3}{2}\cancel{NRA}}{\frac{3}{5}\cancel{NRA}} \Rightarrow \tan\beta = \frac{15}{6} = \frac{5}{2} \Rightarrow \beta = 68.2°$

c) $T\cos\beta = \frac{3}{5}NRA \Rightarrow T = \frac{\frac{3}{5}NRA}{\cos 68.2..} \Rightarrow T = 1.6155...NRA$

$\therefore T = 1.6155... \times \frac{2}{5}W \Rightarrow T = 0.646...W$

$u = 0.65$ (2sf)